$s = r\theta$

$v = \frac{s}{t}$

$w = \frac{\theta}{t}$

$v = \frac{r\theta}{t}$

$v = rw$

$s = rwt$

$s \rightarrow$ distance

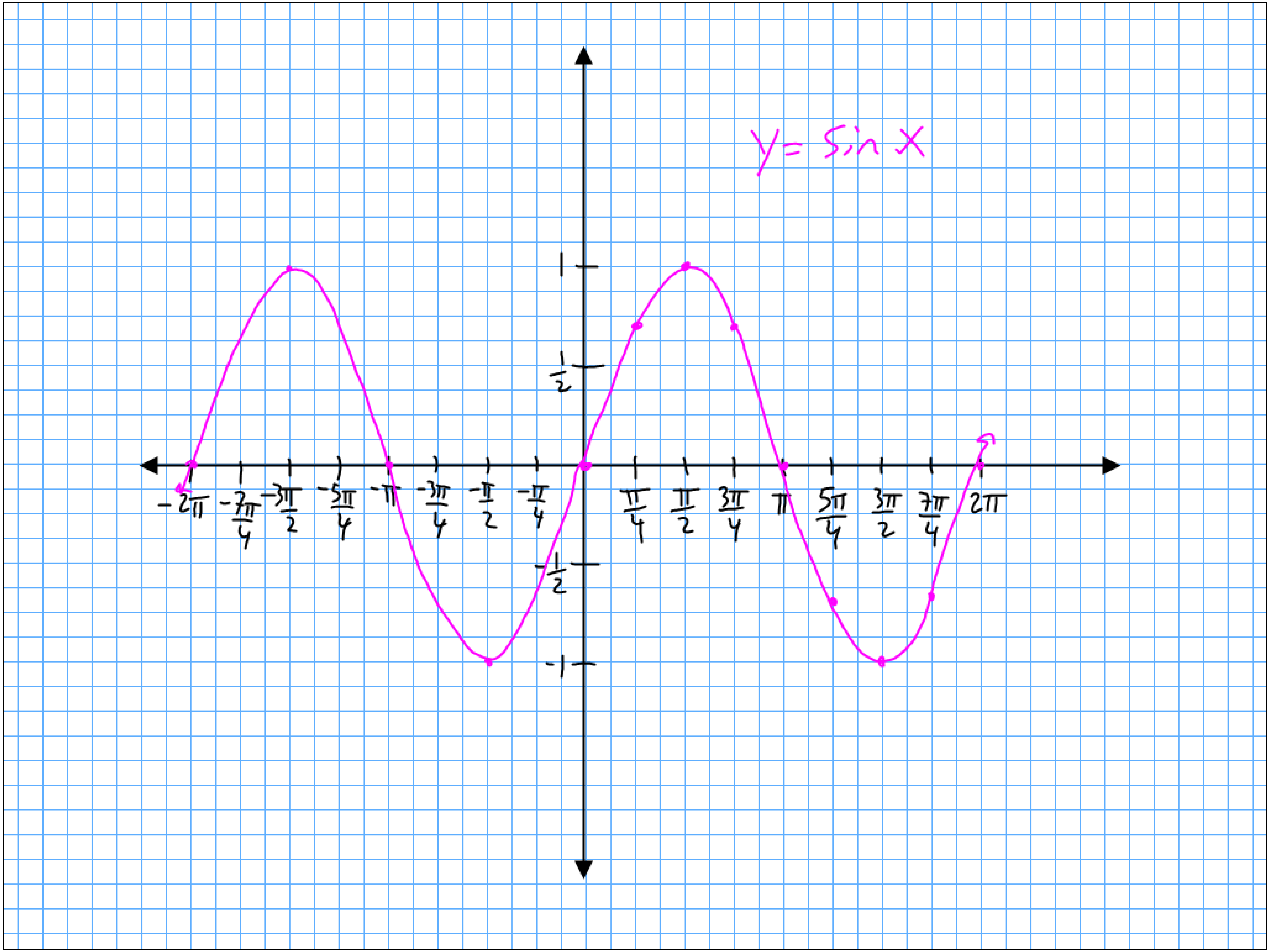
$r \rightarrow$ radius

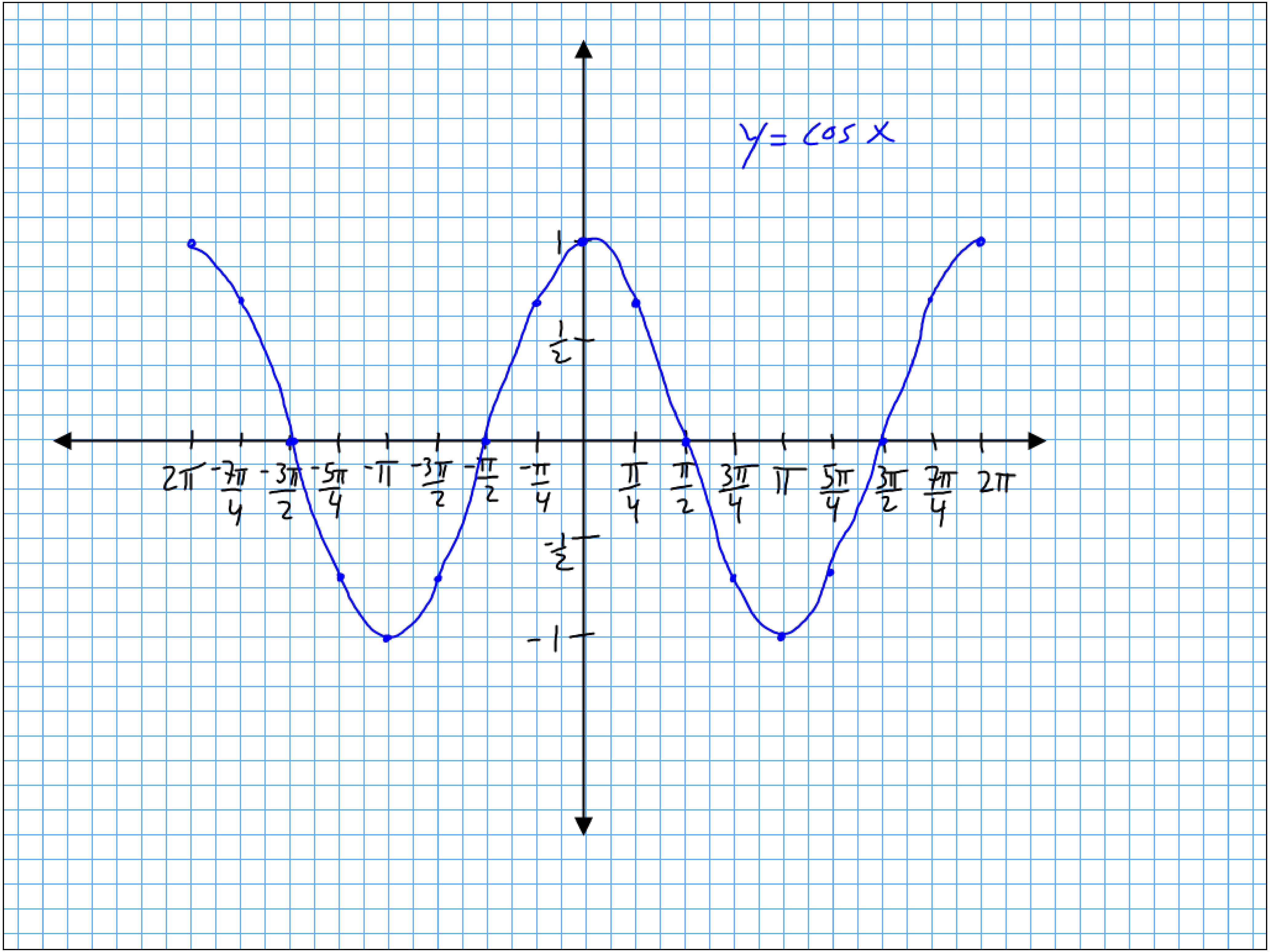
$v \rightarrow$ linear velocity

$w \rightarrow$ angular velocity

$\theta \rightarrow$ angle in radians

$t \rightarrow$ time


y = sin x
1
1/2
-1/2
-1
-2π
-7π/4
-3π/2
-5π/4
-π
-3π/4
-π/2
-π/4
π/4
π/2
3π/4
π
5π/4
3π/2
7π/4
2π


y= cos x
1
1/2
-1/2
-1
2π
-7π/4
-3π/2
-5π/4
-π
-3π/2
-π/2
-π/4
π/4
π/2
3π/4
π
5π/4
3π/2
7π/4
2π

$$y = a\sin\left[b(x-c)\right] + d$$

amplitud (height of bump)

frequency (helps you get period)

horz. shift (beginning endpoint)

vert. shift (this is your midline)

$$y = a\cos\left[b(x-c)\right] + d$$

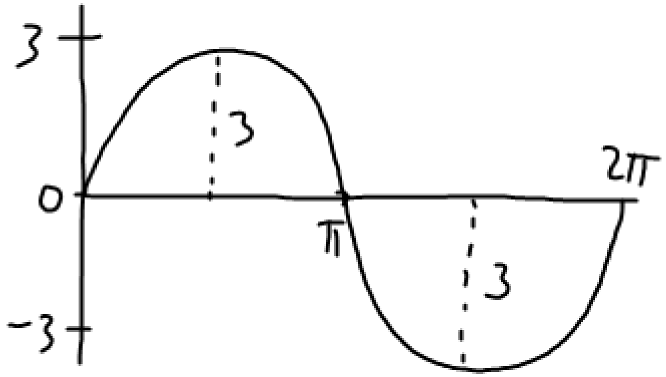
Period – time it takes for 1 complete cycle

$$\text{Period} = \frac{2\pi}{b}$$

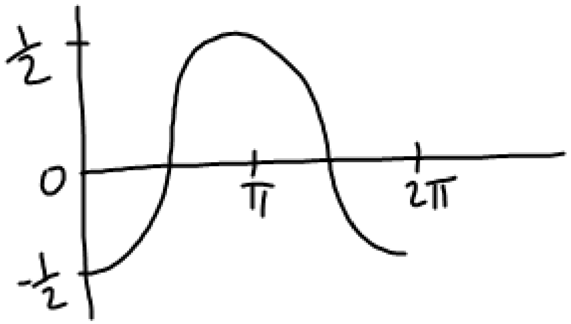
Vertical Stretch (a)

$y = a\sin x$ the a is the amplitude or vertical stretch

$y = 3\sin(x)$



$y = -\frac{1}{2}\cos(x)$



the ½ changes the amplitude, the negative flips the graph over x-axis

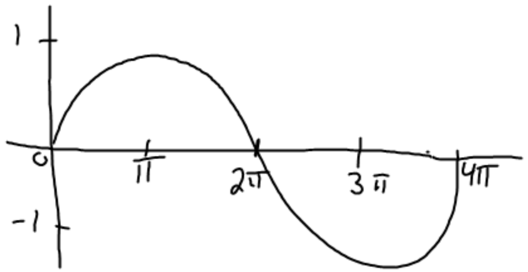
horizontal stretch/compression (b)

$y = \sin bx$

frequency $= \frac{1}{2}$

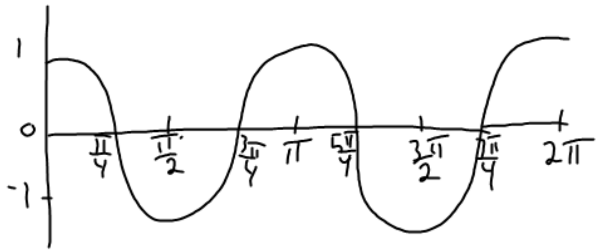
$y = \sin \frac{1}{2}x$ → will do $\frac{1}{2}$ cycle in 2π

Period $= \frac{2\pi}{b} \Rightarrow \frac{2\pi}{\frac{1}{2}} \Rightarrow 2\pi \cdot \frac{2}{1} = 4\pi$



$y = \cos 2x$ frequency is 2 so it will do 2 cycles in 2π

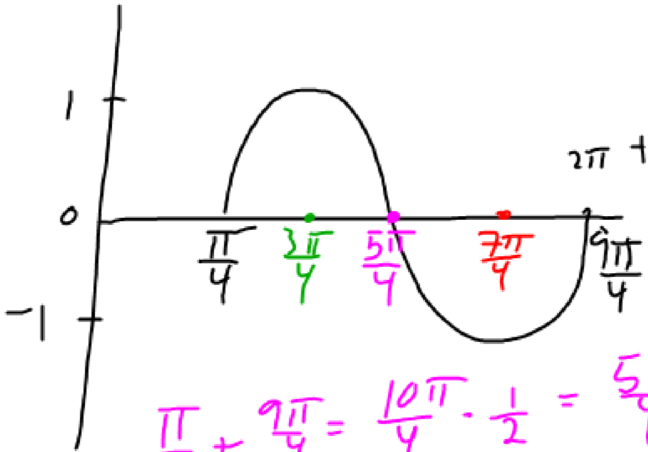
Period $= \frac{2\pi}{2} = \pi$



horizontal shift (c)

$y = \sin(x - c)$

$y = \sin\left(x - \frac{\pi}{4}\right)$ shifts right $\frac{\pi}{4}$, gives you the starting endpoint



$2\pi + \frac{\pi}{4} \rightarrow \frac{8\pi}{4} + \frac{\pi}{4} = \frac{9\pi}{4}$

$\frac{\pi}{4} + \frac{9\pi}{4} = \frac{10\pi}{4} \cdot \frac{1}{2} = \frac{5\pi}{4}$

$\frac{\pi}{4} + \frac{5\pi}{4} = \frac{6\pi}{4} \cdot \frac{1}{2} = \frac{3\pi}{4}$

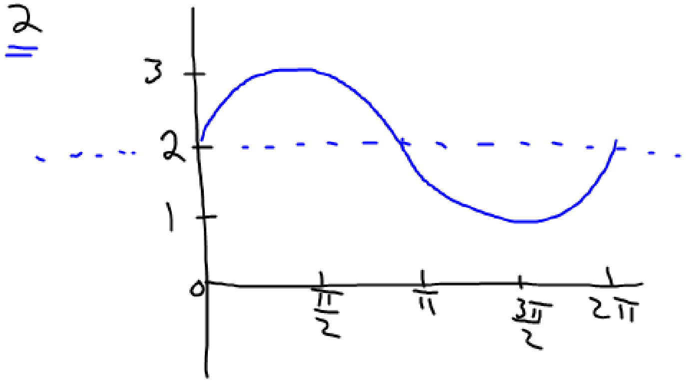
$\frac{5\pi}{4} + \frac{9\pi}{4} = \frac{14\pi}{4} \cdot \frac{1}{2} = \frac{7\pi}{4}$

Vertical shift (d)

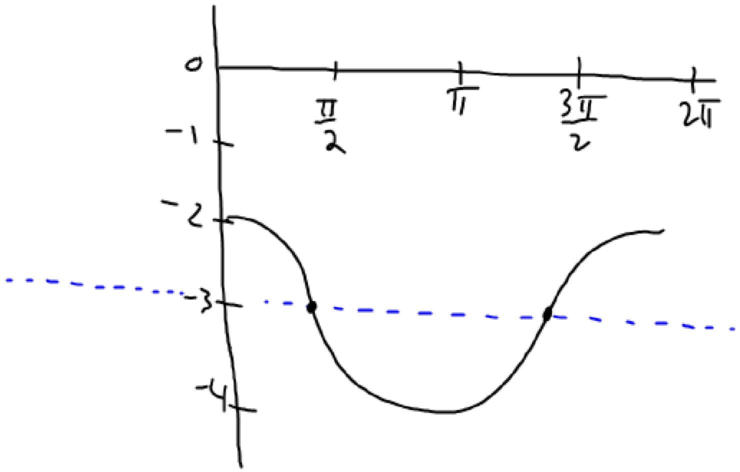
$y = \sin(x) + d$

The d-value gives you the mid-line

$y = \sin(x) + \underline{\underline{2}}$

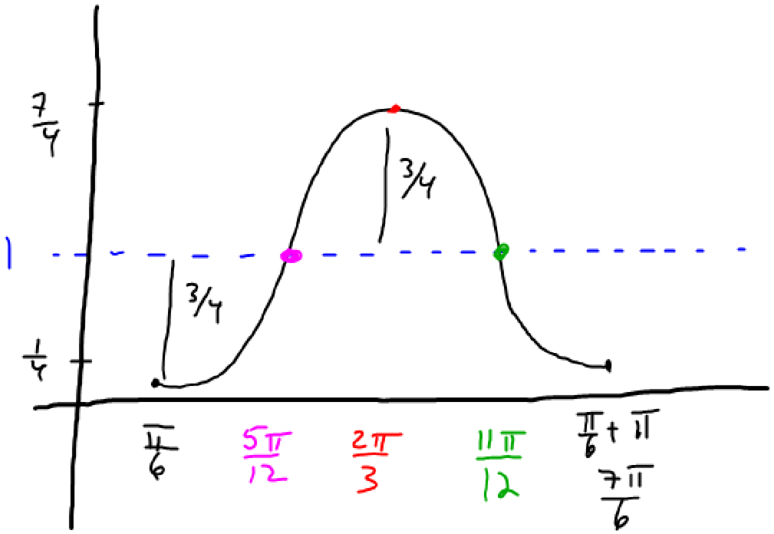


$y = -3 + \cos(x) \rightarrow y = \cos(x) - 3$



Example

$$y = -\frac{3}{4}\cos\left[2\left(x-\frac{\pi}{6}\right)\right]+1$$



$$P = \frac{2\pi}{b} \Rightarrow \frac{2\pi}{2} = \pi$$

$$\frac{\pi}{6}+\frac{7\pi}{6} = \frac{8\pi}{6}\cdot\frac{1}{2} = \frac{4\pi}{6} = \frac{2\pi}{3}$$

$$\frac{\pi}{6}+\frac{2\pi}{3} = \frac{\pi}{6}+\frac{4\pi}{6} = \frac{5\pi}{6}\cdot\frac{1}{2} = \frac{5\pi}{12}$$

$$\frac{2\pi}{3}+\frac{7\pi}{6} = \frac{4\pi}{6}+\frac{7\pi}{6} = \frac{11\pi}{6}\cdot\frac{1}{2} = \frac{11\pi}{12}$$

My Steps

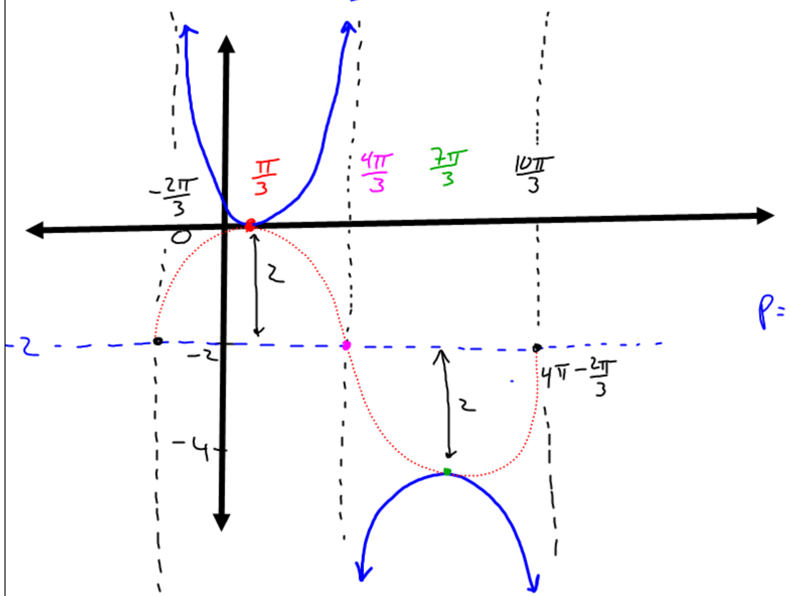
1. Draw curve first
2. Put in midline
3. Find Period
4. Find the horz. shift to get the starting endpoint
5. Find the ending endpoint using the start point plus the period
6. Find x-coord. by avg.
7. Put in axes and find y-coord. using the a-term

Secant + cosecant

- Secant matches up with cosine
- cosecant matches up with sine

⟹ For sec x + csc x graph the reciprocal function and use as a guide for the sec x and csc x. Put in vertical asymptotes wherever the guide crosses midline.

$y = 2\csc\left[\frac{1}{2}\left(x+\frac{2\pi}{3}\right)\right] - 2$ graph $y = 2\sin\left[\frac{1}{2}\left(x+\frac{2\pi}{3}\right)\right] - 2$



$P = \frac{2\pi}{\frac{1}{2}} = 4\pi$

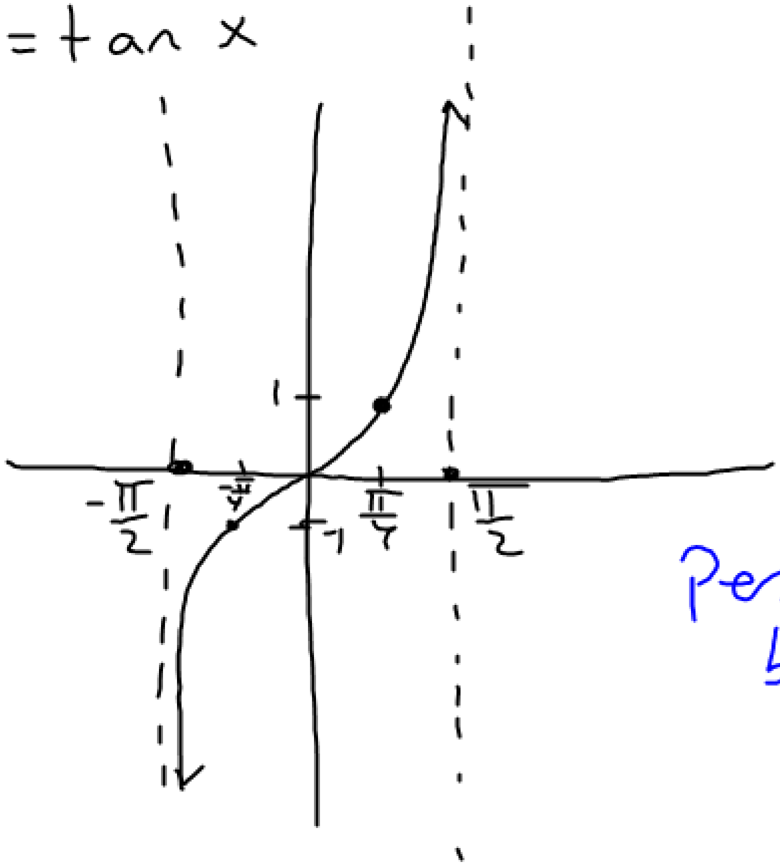
$\frac{-2\pi}{3} + \frac{10\pi}{3} = \frac{8\pi}{3} \cdot \frac{1}{2} = \frac{4\pi}{3}$

$\frac{-2\pi}{3} + \frac{4\pi}{3} = \frac{2\pi}{3} \cdot \frac{1}{2} = \frac{\pi}{3}$

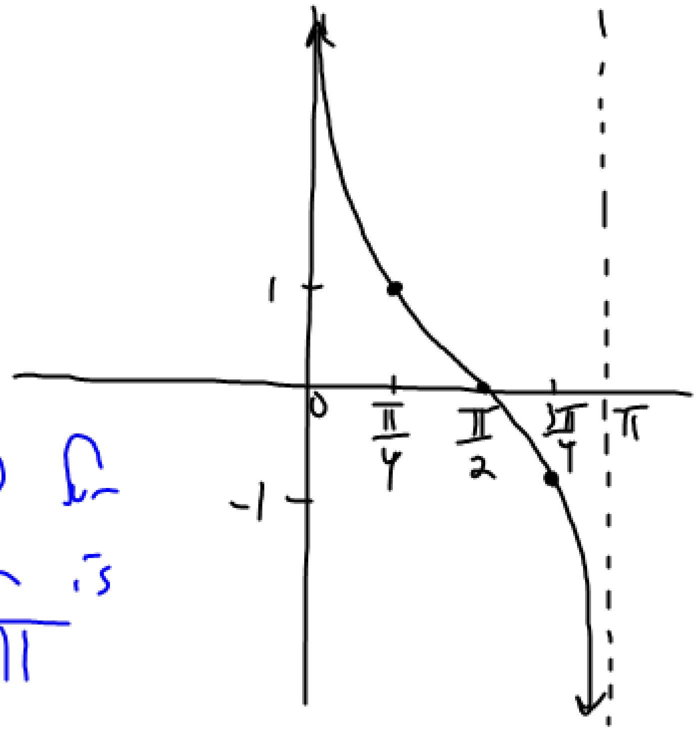
$\frac{4\pi}{3} + \frac{10\pi}{3} = \frac{14\pi}{3} \cdot \frac{1}{2} = \frac{7\pi}{3}$

tangent & cotangent

$y = \tan x$



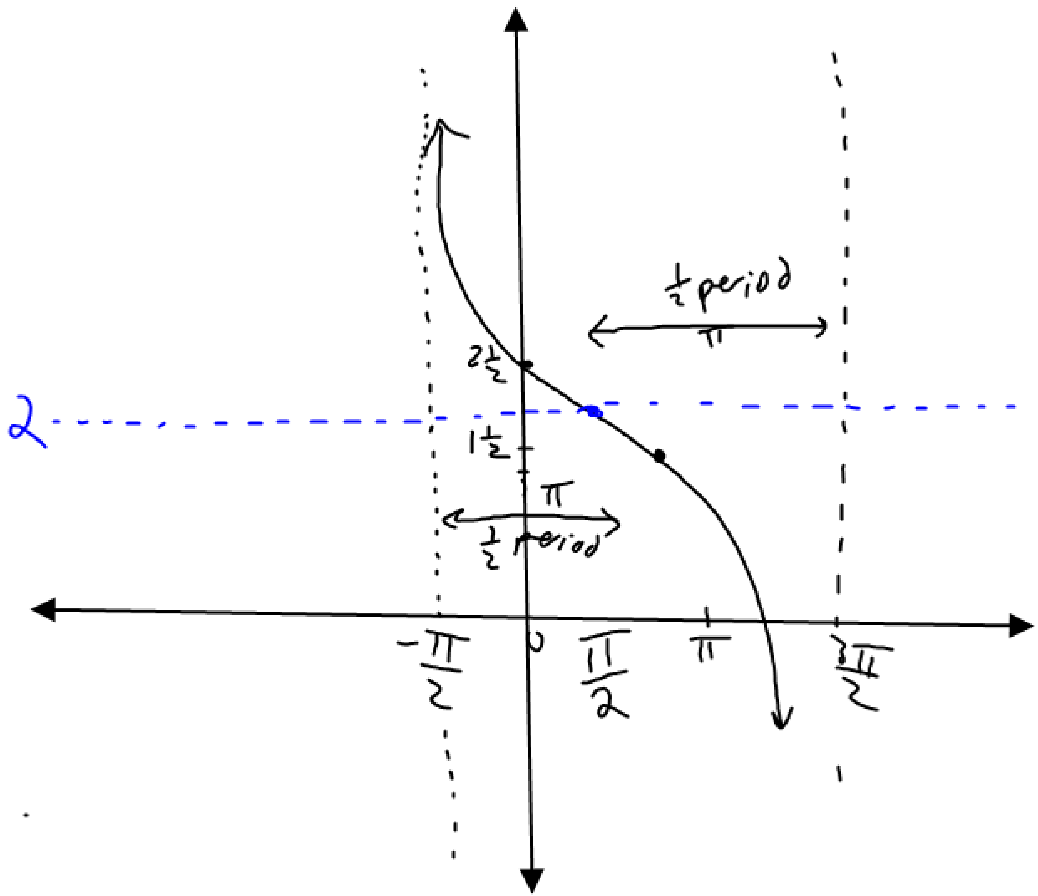
$y = \cot x$



Period for both is π

Example

$$y = -\frac{1}{2}\tan\left[\frac{1}{2}\left(x - \frac{\pi}{2}\right)\right] + 2$$



① sketch curve

② Find period
$p = \frac{\pi}{b}$

③ Find the shift using the center

④ start point is the center $- \frac{1}{2}$ p

⑤ end point is the center $+ \frac{1}{2}$ period

⑥ Put in axes

⑦ $\frac{1}{2}$-way between center and asymptotes
Find y-coord. by doing midline plus and minus a-term

The first step to modeling a periodic function with a sinusoid is to identify the highest and lowest points and the period. You can then use that information to write a function

$y = \pm a\cos\left[b(x-c)\right]+d$ or $y = \pm a\sin\left[b(x-c)\right]+d$.

Example:

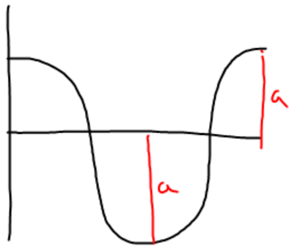
A weight on the end of a spring is pulled down to just 2 inches above the floor and then released. After 0.8 sec. it reaches its highest point 12 inches off the floor. Write a function for the height of the weight from the floor with respect to time.

Analysis:

- The period is the time for 1 complete cycle. In this case it takes 0.8 seconds for $\frac{1}{2}$ a cycle so it will take 1.6 seconds for a complete period.
- The high point is given to us $(0.8, 12)$.
- The low point is the initial condition in this example $(0, 2)$
- The equation would be $y = 5\cos\left[\frac{5\pi}{4}(x-0.8)\right]+7$ using the method on the next slide.

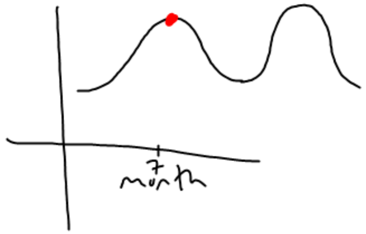
Note: $\frac{2\pi}{1.6} = \frac{2\pi}{\frac{8}{5}} = \frac{10\pi}{8} = \frac{5\pi}{4}$

$$y = a\cos\left[b(x-c)\right] + d$$



$$a = \frac{\underset{(y\text{-value})}{\text{high point}} - \underset{(y\text{-value})}{\text{low point}}}{2}$$

$b = \dfrac{2\pi}{\text{Period}}$, Period from context



c = x-coord. of the high point

$$d = \frac{\underset{(y\text{-coord})}{\text{high point}} + \underset{(y\text{-coord})}{\text{low point}}}{2}$$

PLEASE CHECK
YOUR EQUATION

- Plug it in the y=
- look at the table for known points
- or create a scatter plot with given data and graph your function in the same window to see fit.